NON NEIGHBOURLY EDGE IRREGULAR BIPOLAR FUZZY GRAPHS

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ABSTRACT

This paper discussed non-neighbourly edge irregular bipolar fuzzy graphs and non-neighbourly edge totally irregular bipolar fuzzy graphs. Some of the characteristics of non-neighbourly edge irregular bipolar fuzzy graphs are investigated and tested for non-neighbourly edge totally irregular bipolar fuzzy graphs. We study the edge irregularity on bipolar fuzzy graphs using the underlying graphs cycle, path, comb, star, and bi-star.

Keywords: edge irregular bipolar fuzzy graph, non-neighbourly edge irregular, non-neighbourly edge totally irregular bipolar fuzzy graph.

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Introduction

The fuzzy sets (FS) that Zadeh described in 1965 have demonstrated useful applications in numerous academic domains. Since the Cantorian set was unable to manage uncertainty and ambiguity, the fuzzy set concept is welcomed. R. Jahir Hussain, S. Yahya Mohamed, and A. Nagoorgani presented irregular bipolar fuzzy graphs[4]. The concepts of edge degree and total edge degree were presented by M. G. Karunambigai, S. Siva Sankar, and K. Palanivel, who also studied the degree of an edge in a bipolar fuzzy graph[1]. The neighbourly edge irregular bipolar fuzzy graph was developed by N. R. Santhi Maheswari and C. Sekar [6]. We are encouraged to talk about some of the characteristics of non-neighbourly edge irregular bipolar fuzzy graphs and non-neighbourly edge totally irregular bipolar fuzzy graphs.

Non-Neighbourly Edge Irregular Bipolar

Fuzzy Graphs

Non-neighbourly edge irregular and non-neighbourly edge totally irregular bipolar fuzzy graphs are defined and some of their properties are examined in this section.

Definition 2.1: Let ψ be a connected bipolar fuzzy graph on ψ^*. Then ψ is said to be a non-neighbourly edge irregular bipolar fuzzy graph if each pair of non-adjacent edges has distinct degrees.

Definition 2.2: Let ψ be a connected bipolar fuzzy graph on ψ^*. Then ψ is said to be a non-neighbourly edge irregular bipolar fuzzy graph if each pair of non-adjacent edges has distinct total degrees.

Theorem 2.3: Let ψ be a connected bipolar fuzzy graph on ψ^*. If ψ is a non-neighbourly edge irregular bipolar fuzzy graph then ψ is an edge irregular bipolar fuzzy graph.

Proof. Let ψ be a connected bipolar fuzzy graph on $\psi*$. Assuming that ψ is a non-neighbourly edge irregular bipolar fuzzy graph. Then every two non-adjacent edges in ψ have distinct degrees. Thus there exists an edge that is adjacent to the edges having distinct degrees. Hence ψ is an edge irregular bipolar fuzzy graph.

Theorem 2.4: Let ψ be a connected bipolar fuzzy graph on ψ^*. If ψ is a non-neighbourly edge totally irregular bipolar fuzzy graph, then ψ is an edge totally irregular bipolar fuzzy graph.

Proof. *The proof* is comparable to Theorem 2.3. □

Remark 2.5: The converse of the above-mentioned theorems, 2.3 and 2.4, are not required to be true.

Theorem 2.6: Let ψ be a connected bipolar fuzzy graph on ψ^*, a path on n, $n \geq 3$ vertices. If the membership values of the edges $u_1, u_2, \ldots, u_{n-1}$ are respectively $l_1, l_2, \ldots, l_{n-1}$ and $s_1, s_2, \ldots, s_{n-1}$ like that $l_1 > l_2 > \cdots > l_{n-1}$ and $h_1 > h_2 > \ldots > h_{n-1}$, then ψ is both non-neighbourly edge irregular and non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Let ψ be a connected bipolar fuzzy graph on ψ^*, a path on n vertices. Let $u_1, u_2, \ldots, u_{n-1}$ be the edges of ψ^* in that order. Let the membership values of the edges $u_1, u_2, \ldots, u_{n-1}$ are respectively $(l_1, h_1), (l_2, h_2), (l_3, h_3), \ldots, (l_{n-1}, h_{n-1})$ like that $(l_1, h_1) > (l_2, h_2) > (l_3, h_3) > \ldots > (l_{n-1}, h_{n-1})$. Then

$d_\psi(v_j) = (l_{j-1}, h_{j-1}) + (l_j, h_j)$ if $j = 2, 3, 4, \ldots, n-2$,

$d_\psi(v_1) = (l_1, h_1)$ and

$d_\psi(v_{n-1}) = (l_{n-1}, h_{n-1})$.

$d_\psi(u_j) = (l_{j-1}, h_{j-1}) + (l_{j+1}, h_{j+1})$ if $j = 2, 3, 4, \ldots, n-2$,

$d_\psi(u_1) = (l_2, h_2)$ and

$d_\psi(u_{n-1}) = (l_{n-2}, h_{n-2})$.

There are separate degrees on each of every two non-adjacent edges. □

Therefore, ψ is a non-neighbourly edge irregular bipolar fuzzy graph.

$td_{\psi}(u_j) = (l_{j-1}, h_{j-1}) + (l_{j+1}, h_{j+1}) + (l_j, h_j)$ if $j = 2, 3, 4, \ldots, n-2$,

$td_{\psi}(u_1) = (l_2, h_2) + (l_1, h_1)$ and

$td_{\psi}(u_{n-1}) = (l_{n-2}, h_{n-2}) + (l_{n-1}, h_{n-1})$.

Each pair of non-adjacent edges has a unique total degree.

Thus ψ is a non-neighbourly edge totally irregular bipolar fuzzy graph.

Theorem 2.7: Let ψ be a connected bipolar fuzzy graph on ψ^* a path on n, $n \geq 3$ vertices. If the membership values of the edges $u_1, u_2, \ldots, u_{n-1}$ are respectively $l_1, l_2, \ldots, l_{n-1}$ and $h_1, h_2, \ldots, h_{n-1}$ such that $l_1 < l_2 < \cdots < l_{n-1}$ and $h_1 < h_2 < \ldots < h_{n-1}$, then ψ is both non-neighbourly edge irregular and non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. The result is comparable to Theorem 2.6.

Theorem 2.8: Let ψ be a connected bipolar fuzzy graph on ψ^*, a cycle on m($m \geq 4$) vertices. If the membership values of the edges $u_1, u_2, \ldots, u_m$ are respectively $l_1, l_2, \ldots, l_m$ and $h_1, h_2, \ldots, h_m$ such that $l_1 > l_2 > \ldots > l_m$ and $h_1 > h_2 > \ldots > h_m$, then ψ is both non-neighbourly edge irregular and non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Let ψ be a connected bipolar fuzzy graph on ψ^*, a cycle on m vertices. Let $u_1, u_2, \ldots, u_m$ be the edges of ψ^* in that order. Let the membership values of the edges $u_1, u_2, \ldots, u_m$ are respectively (l_1, h_1), (l_2, h_2), (l_3, h_3), ..., (l_m, h_m) such that $(l_1, h_1) > (l_2, h_2) > (l_3, h_3) > \cdots > (l_m, h_m)$.

$d_{\psi}(v_j) = (l_{j-1}, h_{j-1}) + (l_j, h_j)$ if $j = 2, 3, 4, \ldots, m$,

$d_{\psi}(v_1) = (l_1, h_1) + (l_m, h_m)$.

$d_{\psi}(u_j) = (l_{j-1}, h_{j-1}) + (l_{j+1}, h_{j+1})$ if $j = 2, 3, 4, \ldots, m-1$,

$d_{\psi}(u_1) = (l_2, h_2) + (l_m, h_m)$ and

$d_{\psi}(u_m) = (l_1, h_1) + (l_{m-1}, h_{m-1})$.

Every non-adjacent pair of edges has a different degree.

Thus ψ is a non-neighbourly edge irregular bipolar fuzzy graph.

$td_{\psi}(u_j) = (l_{j-1}, h_{j-1}) + (l_{j+1}, h_{j+1}) + (l_j, h_j)$ if $j = 2, 3, 4, \ldots, m-1$

$td_{\psi}(u_1) = (l_2, h_2) + (l_m, h_m) + (l_1, h_1)$ and

$td_{\psi}(u_m) = (l_1, h_1) + (l_{m-1}, h_{m-1}) + (l_m, h_m)$.

Every non-adjacent pair of edges has a unique total degree.

Therefore ψ is a non-neighbourly edge totally irregular bipolar fuzzy graph.

Theorem 2.9: Let ψ be a connected bipolar fuzzy graph on ψ^*, a cycle on m ($m \geq 4$) vertices. If the membership values of the edges $u_1, u_2, \ldots, u_n$ are respectively $l_1, l_2, \ldots, l_m$ and $h_1, h_2, \ldots$

. , h_m such that $l_1 < l_2 < .... < l_m$ and $h_1 < h_2 < ... < h_m$, then ψ is both non-neighbourly edge irregular and non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. The result is comparable to Theorem 2.8.

Theorem 2.10: Let ψ be a connected bipolar fuzzy graph on ψ^*, a star on $m + 1$ vertices. If the membership values of all the edges are distinct, then ψ is a non-neighbourly edge irregular bipolar fuzzy graph and ψ is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Let $w_1, w_2, . . . , w_m$ be the vertices adjacent to the vertex x. Let $u_1, u_2, . . . , u_m$ be the edges of a star ψ^* in that order have membership values (l_1, h_1) , (l_2, h_2) , (l_3, h_3) , . . . , (l_m, h_m) then

$d_{\psi}(e_j) = (l_1, h_1) + (l_2, h_2) + (l_3, h_3) + \cdots + (l_m, h_m) + (l_j, h_j) - 2(l_j, h_j)$ $(1 \leq j \leq n)$

Every pair of non-adjacent edges, u_j has unique degrees.

Thus ψ is a non-neighbourly edge irregular bipolar fuzzy graph. Also,

$td_{\psi}(u_i) = (l_1, h_1) + (l_2, h_2) + (l_3, h_3) + \ldots + (l_m, h_m) + (l_j, h_j) - 2(l_j, h_j) + (l_j, h_j)$ $(1 \leq j \leq n)$

$td_{\psi}(u_j) = (l_1, h_1) + (l_2, h_2) + (l_3, h_3) + \ldots + (l_m, h_m)$ $(1 \leq j \leq n)$

All the edges u_j $(1 \leq j \leq n)$ have the same number of degrees overall.

Thus, the graph ψ is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

Remark 2.11: Let ψ be a connected bipolar fuzzy graph on ψ^*, a star on m + 1 vertices. If the membership values of any two edges are the same, then ψ is not a non-neighbourly edge irregular bipolar fuzzy graph and G is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

Theorem 2.12: Let ψ be a connected bipolar fuzzy graph on ψ^*, a bi-star graph $B_{n,m}$. If the membership values of all the edges are distinct, then ψ is a non-neighbourly edge irregular bipolar fuzzy graph and ψ is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Let $w_1, w_2, . . . , w_n$ be the vertices adjacent to the vertex a. Let $u_1, u_2, . . . , u_n$ be the edges incident with vertex a in that order have membership values (l_1, h_1) , (l_2, h_2) , (l_3, h_3) , . . . , (l_n, h_n) such that $(l_1, h_1) > (l_2, h_2) > (l_3, h_3) > ... > (l_n, h_n)$.

Let $y_1, y_2, ..., y_m$ be the vertices adjacent to vertex b.

Let $c_1, c_2, ..., c_m$ be the edges incident with vertex b in that order have membership values (p_1, q_1), (p_2, q_2), (p_3, q_3) , . . . , (p_m, q_m) such that $(p_1, q_1) > (p_2, q_2)$

$>(p_3, q_3) > \ldots > (p_m, q_m) > (p, q)$ where (p, q) is the membership value of the edge ab.

Then $d_\psi(ab) = (l_1, h_1) + (l_2, h_2) + \ldots + (l_n, h_n) + (p, q) + (p_1, q_1) + (p_2, q_2) + \ldots + (p_m, q_m) + (p, q) - 2(p, q)$,

$d_\psi(ab) = (l_1, h_1) + (l_2, h_2) + \ldots + (l_n, h_n) + (p_1, q_1) + (p_2, q_2) + \ldots + (p_m, q_m)$

$td_\psi(ab) = (l_1, h_1) + (l_2, h_2) + \ldots + (l_n, h_n) + (p_1, q_1) + (p_2, q_2) + \ldots + (p_m, q_m) + (p, q)$.

$d_\psi(u_j) = (l_1, h_1) + (l_2, h_2) + \ldots + (l_n, h_n) + (p, q) + (l_i, h_i) - 2(l_j, h_j), 1 \leq j \leq n$ and

$d_\psi(h_i) = (p_1, q_1) + (p_2, q_2) + \ldots + (p_m, q_m) + (p, q) + (p_i, q_i) - 2(p_i, q_i), 1 \leq i \leq m$.

There are separate degrees on each of every two non-adjacent edges.

Thus ψ is a non-neighbourly edge irregular bipolar fuzzy graph.

$td_\psi(u_j) = (l_1, h_1) + (l_2, h_2) + \ldots + (l_n, h_n) + (p, q) + (l_j, h_j) - (l_j, h_j), 1 \leq j \leq n$,

$td_\psi(u_j) = (l_1, h_1) + (l_2, h_2) + \ldots + (l_n, h_n) + (p, q), 1 \leq j \leq n$,

$td_\psi(h_i) = (p_1, q_1) + (p_2, q_2) + \ldots + (p_m, q_m) + (p, q) + (p_i, q_i) - (p_i, q_i), 1 \leq i \leq m$,

$td_\psi(h_i) = (p_1, q_1) + (p_2, q_2) + \ldots + (p_m, q_m) + (p, q), 1 \leq i \leq m$.

Observe that all u_j $(1 \leq j \leq n)$ and all c_i $(1 \leq i \leq m)$ have identical total degrees.

Hence ψ is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

Theorem 2.13: Let ψ be a connected bipolar fuzzy graph on ψ^*, a comb on $2n$ vertices. If φ is a constant function then ψ is not a non-neighbourly edge irregular bipolar fuzzy graph and ψ is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Let $u_1, u_2, \ldots, u_{n-1}$ be the edges of the path in ψ^* and $v_1, v_2, \ldots, v_n$ be the corresponding pendant edges in that order. Let φ be a constant function say $\varphi(ab) = (k_1, k_2)$, for all $ab \in E$.

$d_{n_2^+}(u_1) = 2k_1 + 3k_1 - 2k_1 = 3k_1$,

$d_{n_2^-}(u_1) = 2k_2 + 3k_2 - 2k_2 = 3k_2$.

If j = 2, 3, . . . , $n - 2$ then

$d_{n_2^+}(uj) = 3k_1 + 3k_1 - 2k_1 = 4k_1$, $d_{n_2^-}(uj) = 3k_2 + 3k_2 - 2k_2 = 4k_2$,

$d_{n_2^+}(u_{m-1}) = 3k_1 + 2k_1 - 2k_1 = 3k_1$,

$d_{n_2^-}(u_{m-1}) = 3k_2 + 2k_2 - 2k_2 = 3k_2$ and

$d_{n_2^+}(v_1) = k_1 + 2k_1 - 2k_1 = k_1$, $d_{n_2^-}(v_1) = k_2 + 2k_2 - 2k_2 = k_2$.

If j = 2, 3, . . . , $n - 1$ then

$d_{n_2^+}(vj) = k_1 + 3k_1 - 2k_1 = 2k_1$,

$d_{n_2^-}(v_1) = k_2 + 3k_2 - 2k_2 = 2k_2$ and

$d_{n_2^+}(v_n) = k_1 + 2k_1 - 2k_1 = k_1$, $d_{n_2^-}(v_n) = k_2 + 2k_2 - 2k_2 = k_2$.

The degree of the two non-adjacent edges u_1 and u_{m-1} is the same.

Thus ψ is not a non-neighbourly edge irregular bipolar fuzzy graph.

We have $td(ab) = d(ab) + \varphi(ab)$

$td_{n_2^+}(u_1) = 3k_1 + k_1 = 4k_1$, $td_{n_2^-}(u_1) = 3k_2 + k_2 = 4k_2$.

If $j = 2, 3, \ldots, n-2$ then

$td_{n_2^+}(u_j) = 4k_1 + k_1 = 5k_1$, $td_{n_2^-}(u_j) = 4k_2 + k_2 = 5k_2$,

$d_{n_2^+}(u_{n-1}) = 3k_1 + k_1 = 4k_1$, $td_{n_2^-}(u_{n-1}) = 3k_2 + k_2 = 4k_2$ and

$td_{n_2^+}(v_1) = k_1 + k_1 = 2k_1$, $td_{n_2^-}(v_1) = k_2 + k_2 = 2k_2$.

If $j = 2, 3, \ldots, n-1$then

$td_{n_2^+}(v_j) = 2k_1 + k_1 = 3k_1$, $td_{n_2^-}(v_j) = 2k_2 + k_2 = 3k_2$ and

$td_{n_2^+}(v_n) = k_1 + k_1 = 2k_1$, $td_{n_2^-}(v_n) = k_2 + k_2 = 2k_2$.

The two non-adjacent edges u_1 and u_{n-1} have the same total degree.

Hence ψ is not a non-neighbourly edge totally irregular bipolar fuzzy graph.

3. Edge Irregular and Edge Irregular Bipolar Fuzzy Graphs on a Path, Cycle, Comb, Star and Bi-star Graph

This section examines the properties of edge irregular and edge totally irregular bipolar fuzzy graphs on a path, cycle, comb, star, and bi-star graph.

Theorem 3.1: Let ψ be a connected bipolar fuzzy graph on ψ^*, a path on n, $n \geq 3$ vertices. If the membership values of the edges $u_1, u_2, \ldots, u_{n-1}$ are respectively $l_1, l_2, \ldots, l_{n-1}$ and $h_1, h_2, \ldots, h_{n-1}$ such that $l_1 > l_2 > \cdots > l_{n-1}$ and $h_1 > h_2 > \ldots > h_{n-1}$, then ψ is both edge irregular and edge totally irregular bipolar fuzzy graph.

Proof. Theorems 2.3, 2.4, and 2.6 give us ψ is both edge irregular and edge totally irregular bipolar fuzzy graph.

Theorem 3.2: Let ψ be a connected bipolar fuzzy graph on ψ^*, a cycle on m ($m \geq 4$) vertices. If the membership values of the edges $u_1, u_2, \ldots, u_m$ are respectively $l_1, l_2, \ldots, l_m$ and $h_1, h_2, \ldots, h_m$ such that $l_1 > l_2 > \ldots > l_m$ and $h_1 > h_2 > \ldots > h_m$, then ψ is both edge irregular and edge totally irregular bipolar fuzzy graph.

Proof. Theorems 2.3, 2.4, and 2.8 lead to the conclusion that the ψ is an edge irregular and edge totally irregular bipolar fuzzy graph.

Theorem 3.3: Let ψ be a connected bipolar fuzzy graph on ψ^*, a star on $m + 1$ vertices. If the membership values of all the edges are distinct then ψ is an edge irregular bipolar fuzzy graph and ψ is not an edge totally irregular bipolar fuzzy graph.

Proof. We obtain that ψ is an edge irregular bipolar fuzzy graph and ψ is not an edge totally irregular bipolar fuzzy graph from Theorems 2.3, 2.4, and 2.10.

Theorem 3.4: Let ψ be a connected bipolar fuzzy graph on ψ^*, a bi-star graph on $B_{n,m}$. If the membership values of all the edges are distinct, then ψ is an edge irregular bipolar fuzzy graph and ψ is not an edge totally irregular bipolar fuzzy graph.

Proof. According to Theorems 2.3, 2.4, and 2.12, we can conclude that ψ is an edge irregular bipolar fuzzy graph and ψ is not an edge totally irregular bipolar fuzzy graph.

Theorem 3.5: Let ψ be a connected bipolar fuzzy graph on ψ^*, a comb on $2n$ vertices. If φ is a constant function then ψ is not an edge irregular bipolar fuzzy graph and ψ is not an edge totally irregular bipolar fuzzy graph.

Proof. Theorems 2.3, 2.4, and 2.13 lead to the conclusion that ψ is not an edge irregular bipolar fuzzy graph and ψ is not an edge totally irregular bipolar fuzzy graph.

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